Respect the Mathematics You Teach

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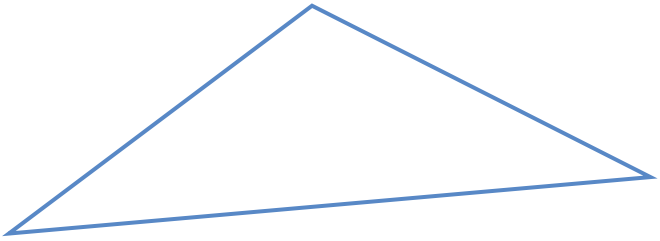


A teacher can never truly teach
unless he is still learning himself.
A lamp can never light another lamp
unless it continues to burn its own flame.
The teacher who has come to the end of his subject,
who has no living traffic with his knowledge
but merely repeats his lessons to his students,
can only load their minds; he cannot quicken them.
–Rabindranath Tagore

During my early years of teaching, Frank Hawthorne was the New York State Supervisor of Mathematics. I had met him when he gave a talk about the state Regents exams to a group in Rochester. Much to my surprise, he remembered me from that brief introduction and he stopped me at an annual conference in Albany the following year. "I have a problem here that I would like you to solve," he told me, and he sketched an obtuse triangle on a sheet of paper. "**All you have to do**," he said, "**is draw line segments that separate the interior of that triangle into acute triangles**," and off he went apparently to offer others the same challenge.

I caught up with Hawthorne a few minutes later to submit my solution. (I don't offer it here because I want you to attempt the exercise yourself.) He was obviously impressed. "I've been giving that problem to dozens of mathematics teachers and very few of them have solved it quickly," and he added, "Many of them couldn't solve it at all."

Hawthorne went on to suggest how little prior knowledge was required to understand the challenge that exercise offered: "All you need to know is what are acute and obtuse angles, acute and obtuse triangles. Those are definitions known to third or fourth grade students."

I cannot resist submitting here the answer one high school teacher gave me to that problem. "What you do," he said, "is draw a segment to divide the obtuse angle into two acute angles. Then you repeat that for the resulting obtuse angle and continue that process over and over. That way the obtuse triangle gets smaller and smaller. When it is infinitely small, you have solved the exercise."

For over sixty years that problem has served to remind me that mathematics can be made challenging at any level. Sadly, I believe that many school mathematics teachers do not think that way and as a result do not respect the content they are teaching. Are you one of those?

Your experience in college encourages this lack of respect. You have studied "higher" mathematics and your advanced studies have made your school math content seem trivial. Factoring quadratic trinomials and applying logarithms are beneath your interest after you have studied field theory and isomorphisms. But if school math is so trivial, how did you do answering that elementary school exercise?

When I taught mathematics teaching methods classes for prospective teachers, my first assignment was always to answer any 10 of the 25 questions on the current American Mathematics Competitions 12 exam. The students always came away from that assignment with more respect for the subject they would

be teaching. There are wonderful questions on these exams, questions designed by serious mathematicians. You will do yourself a favor if each year you challenge yourself by answering that year's questions. Here are just two of the easier exercises from the 2013 test:

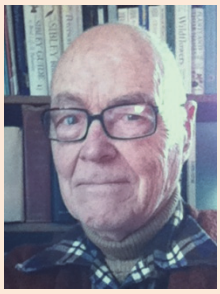
1. A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 runs, not necessarily in that order. They lost by one run in exactly five games. In each of the other games, they scored twice as many runs as their opponents. How many total runs did their opponents score? (A) 35 (B) 40 (C) 45 (D) 50 (E) 55

2. Find the value of:
$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}.$$
(A) -1 (B) 1 (C) 5/3 (D) 30 (E) 35

If you find those exercises not especially difficult, I encourage you to try one of the recent exams collected at www.artofproblemsolving.com. (Note that these exercises also suggest that multiple choice questions can be challenging, something rarely found on standardized tests.)

I want to be careful not to overwhelm you here. The questions on these exams are indeed challenging and many of the more demanding ones do require serious thought. In fact, they require exactly the kind of thoughtful problem solving that you have rarely met in any course: school or college.

It is important for you to understand, however, that you can be a successful mathematics teacher without being able to score well on such exams. To do well on them requires not only depth of understanding but also much practice. And there are many outstanding mathematicians who could do no better than you at solving these problems. Although a few mathematicians, like my former colleague Murray Klamkin, center their careers around developing and solving challenging problems and teaching others to do so; they are in a distinct minority. This represents only a minor aspect of mathematical activity. What these problems do suggest strongly, however, is that the school math you teach is worthy of your respect. I urge you to maintain a high regard for your subject and to communicate that respect to your students. It will make your teaching far more satisfying.



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Both are deeply concerned with classroom instruction of mathematics at all levels and have spoken at state and national meetings on this topic. This essay is a revised excerpt from their new book, *Letters to a Young Math Teacher*, which is designed to help beginners meet the real world of contemporary schools and make the difficult transition from student to teacher, from adolescent to adult. It is available from Amazon.com and other sources. Those who prepare teachers may identify their role to obtain a review copy from the senior author at insrisg@buffalo.edu.